Thermal behaviour of buildings under random conditions

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This paper discusses the use of a building thermal analysis methodology in which the stochastic nature of the forcing functions is considered. This method provides a rational and convenient way of handling uncertainty in the analysis and design of the thermal system. A set of stochastic differential equations is used to model the thermal behaviour of the building. Randomness in the inputs as well as in the coefficients of the model are considered simultaneously. Randomness in this case is modelled as a Gaussian white noise, which provides the mean and standard deviation of the variables. The method is applied to a small-scale problem for verification purposes. The procedure is also applied to the study of thermal behaviour of a direct-gain solar room in which the internal and external forcing functions are considered to be random.

Keywords: stochastic differential equation, moment equations, thermal network, building thermal behavior

Introduction

Prediction of the long-term energy performance of buildings is an essential element in the design of the building and of the heating system components. The thermal performance of a building is a function of solar radiation, ambient temperature, and wind speed, among other input parameters. Several simulation programs have been developed to calculate the long-term energy performance of buildings.[1-3] These simulation programs are the principal predictive tools used to design, renovate, or improve new or existing buildings. The input to these programs is historical hourly weather data, available on magnetic tape. The programs, however, do have limitations. Specifically, they use data that may belong to a "typical" year; therefore the output does not represent the actual long-term performance. Often, data for several years are required. This necessitates considerable data handling on a digital computer.

Another major problem with the programs is that they model the physical systems deterministically, which implies that the spread of the input parameter values is zero and that we are 100% certain of the corresponding output values. The design value must then be multiplied by a safety factor to take account of the uncertainty in the input variables and parameters. The safety factor is based on professional experience. The deterministic simulation is valid if the effect of fluctuations in the forcing functions (radiation, wind, temperature, etc.) is negligible when compared to the mean value. When random fluctuations of the forcing functions are significant, the variance of response can no longer be neglected, as, for example, for a passive solar building where a portion of, or all of, the required load is supplied by solar light entering the room through the windows.

As an alternative to the use of hourly historical weather data, statistical methods have been developed

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to generate synthetic weather data.[4,5] Studies have shown that the quality of the generated data using these methods varies widely.[6] Further critique and evaluation of these models may be found in the literature.[7,8]

Lameiro and Duff[9] developed a Markov chain model to evaluate the long-term expected performance of a solar heating system. The model gives the probabilities of transition from a given level of ambient temperature, solar radiation, and load to other levels of these variables, taking into account the present state of the system. Paassen[5] used the same procedure to evaluate the thermal load of buildings. Anand *et al.*[10] used a probabilistic method to predict the performance of a solar cooling system. They developed an algorithm that generates a probabilistic matrix (temperature and solar radiation has an associated probability) and an analytical formulation that generates synthetic weather data.

The various procedures noted above predict only the expected behaviour of the state of the system. That is, these procedures are able to give the single-point probability information (single statistic, the expected value of response). The other limitation is that the parameters defining the physical system (convective heat transfer coefficients) are considered to be deterministic. These parameters, which may vary randomly, are not included in these models. Finally, they are not capable of describing all types of uncertainties in the system.

Shortcut methods such as solar load ratio (SLR)[11,12] have been developed for fixed configurations and give results that are valid for a limited range of parameters. The SLR correlation is obtained by a statistical least-square fit to a family of monthly performance curves of a particular passive solar system. The drawback of this method is that the effects of all potential hourly and daily fluctuations are smoothed out, and the user is restricted to a "reference" house. Therefore the essential physics of the problem is lost.

In this study a procedure is presented to predict the dynamic performance of buildings. This procedure includes a consideration of the stochastic behaviour of the forcing function.[13–15]

The energy balance equation thereby reduces to a stochastic differential equation, the solution of which represents the performance characteristic. In a stochastic model of a physical system the principal objective is to establish the relationship between the statistics of the response of the model and the statistics of the random input variables and parameters of the model. The results of the analysis reported here can be considered to provide complementary information (safety factor) to those derivable from traditional deterministic methods, so the two techniques may share their strengths while overcoming each other's weaknesses.

The stochastic method described here also helps to simplify the data-handling procedure, in the sense that the processing of the weather data need be done only once to obtain the statistical properties of the input data. Thus, this method could use several years of data as a base to provide a realistic assessment of long-term performance that is preferable to that provided by simulation programs with a few years of real weather data.

Model development

A technique often employed to characterize the heat flow through buildings is to approximate the actual distributed temperatures within the building and its materials by a finite set of nodes at which temperatures are calculated. An energy balance equation can then be written for each node containing terms that show heat flow from node to node by conduction, convection, or radiation; heat flow into a node from internal gains or solar gains; and heat storage within the mass of material represented by the node. In general, the energy balance can be rewritten as state equations:

$$\frac{dT}{dt} = f(T, t) \tag{1}$$

where T is the n-vector of temperature and $f(T, t)$ is a vector of functions that quantitatively represents the rate of change in temperature with respect to time. The solution to this differential equation for appropriate boundary conditions can be found by ordinary calculus. The following is an example that highlights the approach and helps to illustrate the advantage of the method.

Consider a wall section for which the height and the width are much greater than the thickness (one-dimensional heat flow assumption). The wall is discretized into two slices. Lumped-parameter heat balances on each slice are as follows:

$$\begin{aligned} c\frac{dT_1}{dt} &= h_{co}(T_o - T_1) + k(T_2 - T_1) + Q_o \\ c\frac{dT_2}{dt} &= h_{ci}(T_i - T_2) + k(T_1 - T_2) \end{aligned} \tag{2}$$

where T_o and T_i are the outside and inside air temperatures, respectively; h_{ci} and h_{co} are the inside and outside film coefficients, respectively; c is the thermal storage; Q_o is the solar radiation absorbed by the exterior surface of the wall; T_1 is the exterior surface temperature of the wall; T_2 is the interior surface temperature of the wall; and k is the thermal conductance of the wall.

Equation (1) represents a deterministic system that assumes there is no uncertainty in either the parameter values or the inputs, and hence the output can be predicted with certainty. This equation cannot adequately describe many physical systems, because the system parameters are known only in terms of their distributions. For example, the input parameters in equation (2) are stochastic in nature.[4] It is then possible to include the randomness by the equations

$$h_{co} = \bar{h}_{co} + h'_{co}, \quad Q_o = \bar{Q}_o + Q'_o, \quad T_o = \bar{T}_o + T'_o \tag{3}$$

where the overbarred quantities are the deterministic parts that may or may not be time dependent. The primed parts are random noise (stochastic fluctuations).

Stochastic characteristics arise primarily because the environment in which the physical system is embedded has numerous degrees of freedom for oscillation, causing the physical input processes from the environment to the system to fluctuate rapidly. Thus, for realistic modelling, the state equation has to be rewritten as a vector stochastic differential equation of the form

$$\frac{dT}{dt} = f(T, t) + G(T, t)n(t) \tag{4}$$

where $n(t)$ is a noise term and $G(T, t)$ is a function denoting the sensitivity of the system to the noise term. Substituting equation (3) in equation (2), we can write the vectors $T, f(T, t)$ and $G(T, t)$ as

$$f(T, t) = \begin{vmatrix} \frac{\bar{h}_{co}}{c}(\bar{T}_o - T_1) + \frac{k}{c}(T_2 - T_1) + \frac{\bar{Q}_o}{c} \\ \frac{h_{ci}}{c}(T_i - T_2) + \frac{k}{c}(T_1 - T_2) \end{vmatrix}$$

$$T^t = | T_1 \quad T_2 | \tag{5}$$

$$G(T, t) = \begin{vmatrix} \frac{\bar{T}_o - T_1}{c} & \frac{\bar{h}_{co}}{c} & \frac{1}{c} \\ 0 & 0 & 0 \end{vmatrix}$$

$$n(t) = \begin{vmatrix} h'_{co} & 0 & 0 \\ 0 & T'_o & 0 \\ 0 & 0 & Q'_o \end{vmatrix}$$

The product term $h_{co} T_o$ has been linearized as follows:

$$\begin{aligned} h_{co} T_o &= (\bar{h}_{co} + h'_{co})(\bar{T}_o + T'_o) \\ &= \bar{h}_{co} \bar{T}_o + \bar{h}_{co} T'_o + h'_{co} \bar{T}_o \end{aligned}$$

The term $h'_{co} T'_o$ is neglected because it is second order, the product is small compared to the mean values, and the resulting equation is then linear and soluable.

The random-noise term should have two features: (1) It should be sufficiently comprehensive to adequately describe the random disturbance. (2) It should provide for the existence and uniqueness of the solution. In general, white noise is not only able to describe the random fluctuations in the environment in a physically meaningful way, but it is also able to give a unique and satisfactory solution. The solution is developed based on the Ito calculus.[16–18]

To solve equations (4), we make the useful and reasonable assumption that the stochastic noise is a wideband process with a flat frequency spectrum up to very high frequencies. Such wideband processes are almost delta-correlated so that they can be approximated by white-noise processes. Further, these stochastic noise terms occur because of the many highly irregular, rapidly fluctuating random functions whose total effect, according to the central limit theorem, obeys the Gaussian law. Thus, $n(t)$ is considered to be a white Gaussian process. This is the usual engineering approach to white noise $W(t)$ as a zero-mean, wide-sense stationary process defined by

$$\begin{aligned} &\langle W(t) \rangle = 0 \\ &\langle W(t) \quad W(t-s)^T \rangle = D(T)\delta(t) \end{aligned} \tag{6}$$

where $\langle \cdot \rangle$ denotes expectation, $D(t)$ is the diagonal covariance parameter matrix (which shows how $W(t)$ is correlated with itself), and $\delta(t)$ is the Dirac delta function. Mathematically this implies that the white-noise process has zero correlation at time $t = s$ and infinite variance. The solution to equations (4) cannot be obtained by direct integration because white-noise processes are not ordinary functions of time.

Stochastic differential equation (SDE)

The white Gaussian process is described as the formal derivative in time of the Brownian motion process with independent increments dB in time dt characterized by

$$\langle dB(t) \rangle = 0$$

and (7)

$$\langle dB(t) \quad dB(t)^T \rangle = D(T)\, dt$$

where

$$dB(t) = W(t)\, dt$$

The Brownian motion process is not differentiable, because for time $t_0 < t_1 < t_2$, the increments $B(t_2) - B(t_1)$ and $B(t_1) - B(t_0)$ are independent of each other, no matter how small the differences $t_2 - t_1$ and $t_1 - t_0$ are.

Now, equation (4) can be rewritten as

$$dT = f(T, t)\, dt + G(T, t)\, dB(t) \tag{8}$$

where $f(T, t)$, $i = 1, \ldots, n$ and $G(T, t)$ are given continuous functions having continuous second-order derivatives with regard to all variables for $T \in R$ and $B(t_i)$ are independent Wiener processes. The stochastic differential equation (8) is interpreted to be equivalent to the stochastic integral equation

$$T(t) - T(t_0) = \int_{t_0}^{t} f(T, s)\, ds + \int_{t_0}^{t} G(T, s)\, dB(s) \tag{9}$$

The first integral on the right side of equation (9) can be defined under suitable conditions as a mean-square Riemann integral or as an ordinary integral for specified sample functions. The second integral cannot be defined as a mean-square Riemann–Stieltjes integral.[16–18]

Ito[19] introduced an interpretation for stochastic integrals that enables the solution process $T(t)$ of equation (9) to be considered as a diffusion Markov process. This solution can be obtained by Ito calculus. Another representation of stochastic integrals associated with the differential equation was developed by Stratonovitch.[17] He treated diffusion Markov processes as smooth random functions of time. Using the Stratonovitch definition, one can manipulate stochastic integrals and differential equations in the same way as Riemann–Stieltjes integrals and ordinary differential equations.

Stochastic integrals and differential equations of Ito and Stratonovitch

Consider the n-vector integral of the form

$$I = \int G(T, t)\, dB(t) \tag{10}$$

where $\{B(t), t \in T\}$ is an m-vector Wiener process, and $G(T, t)$ is an $n \times m$ matrix function of its arguments. As stated previously, this stochastic integral cannot be defined as a mean-square Riemann–Stieltjes integral because the Brownian motion process is a mathematical artifice. Although it is mean-square continuous, it is not differentiable anywhere. Therefore, an interpretation is necessary in evaluating this integral.

Let (t_{k-1}, t_k) be a subinterval of (a, b), and let t'_k be an arbitrary point in the range (t_{k-1}, t_k) where the integrand is specified. The integral can be defined as the limit of Riemann sums as follows:

$$I_n = G(T, t_k)(B(t_k) - B(t_{k-1})) \tag{11}$$

Then

$$\int_a^b G(T, t)\, \mathrm{d}B = \underset{\substack{n\to\infty \\ D_n \to 0}}{\mathrm{l.i.m}}\, I_n \tag{12}$$

where l.i.m is the limit in the mean and

$$D_n = \max(t_k, t_{k-1}) \tag{13}$$

Because of the particular mathematical nature of the Brownian motion process considered here, the value of the above limit depends on the point in the interval where the integrand $G(T, t_k)$ is evaluated.

With the stochastic integral defined as an Ito integral, the integrand is specified at the beginning of the time interval for the time step (t_{k-1}, t_k). Therefore, the expected value of the Ito stochastic becomes zero, because of the independence of the integrand and the Brownian motion increment. Therefore, with

$$I_0 = \sum_{i=1}^{n} G(T, t_i)(B(t_{i+1}) - B(t_i)) \tag{14}$$

we have

$$\int_a^b G(T, t)\, \mathrm{d}B = \underset{\substack{n\to\infty \\ D_n \to 0}}{\mathrm{l.i.m}}\, I_0 \tag{15}$$

where

$$\langle I_0 \rangle = 0 \tag{16}$$

In the Stratonovitch procedure the integrand is defined at the symmetric midpoint for the time step (t_{k-1}, t_k), and the integral is called the Stratonovitch integral.[16] With

$$I_{1/2} = \sum_{i=1}^{n} G\left(\frac{T(t_{i+1}) - T(t_i)}{2}, \frac{t_{i+1} + t_i}{2}\right) \times (B(t_{i+1}) - B(t_i)) \tag{17}$$

we have

$$\int G(T, t)\, \mathrm{d}B = \underset{\substack{n\to\infty \\ D_n \to 0}}{\mathrm{l.i.m}}\, I_{1/2} \tag{18}$$

If we denote the Stratonovitch integral (Stratonovitch symmetrized stochastic integral) by I and the Ito stochastic integral by I_0, the relation between the Stratonovitch integral and the Ito integral is given by[17,20]

$$I_{1/2} = \int_a^b G(T, t) D(t) \frac{\partial G^{\mathrm{T}}(T, t)}{\partial B}\, \mathrm{d}t + I_0 \tag{19}$$

where $D(t)$ is defined by equation (4).

The first integral on the right side is a mean-square Riemann integral. Depending on the choice of integral, we may define either an Ito or a Stratonovitch SDE; the process specified by an SDE of one type can be identically specified by an SDE of the other form.

If in equation (8), $G(T, t)$ is independent of T, the equation can be interpreted as an Ito equation or as a Stratonovitch equation. This is evident from the relationship (19) between the two stochastic integrals.

In what follows, the SDE is interpreted in the Ito sense. The moment equations will be generated by an equivalent Ito equation.

Ito differential rule

Consider an n-vector SDE of the form

$$\mathrm{d}T = f(T, t)\, \mathrm{d}t + G(T, t)\, \mathrm{d}B(t) \tag{20}$$

$$T(t_0) = T_0, \qquad t_0 < t < T$$

where $B(t)$ is the m-vector Wiener process and $T(t)$ is an n-vector stochastic process.

A lemma in stochastic Ito calculus[16] states that if θ is a scalar-valued real function of the solution process to the SDE (20) such that it has continuous first and second derivatives, then, with reference to SDE (20), the SDE satisfied by θ is

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \sum_{j=1}^{n} f_j(T, t) \frac{\partial \theta}{\partial T_j} + \frac{1}{2} \sum_{i,j=1}^{n} (GDG^{\mathrm{T}})_{ij} \frac{\partial^2 \theta}{\partial T_i\, \partial T_j} + \frac{\partial \theta}{\partial t} \tag{21}$$

Taking the expectation of equation (21) yields

$$\frac{\mathrm{d}\langle\theta\rangle}{\mathrm{d}t} = \sum_{j=1}^{n} \left\langle f(T, t) \frac{\partial \theta}{\partial T_j} \right\rangle + \frac{1}{2} \sum_{i,j=1}^{n} \left\langle (GDG^{\mathrm{T}})_{ij} \frac{\partial^2 \theta}{\partial T_i\, \partial T_j} \right\rangle + \left\langle \frac{\partial \theta}{\partial t} \right\rangle \tag{22}$$

Let

$$\theta = \prod_{i=1}^{n} T_i^{k_i} \tag{23}$$

where the k_i are positive integers satisfying $\sum_{i=1}^{n} k_i = m$, where m is the order of the moment and n is the number of variables. Substituting equation (23) into equation (22) gives the mixed moments $\langle T_1 \quad T_2 \quad \cdots \quad T_n \rangle$ of order k_n:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\langle \left(\prod_{i=1}^{n} T_i^{k_i} \right) \right\rangle = \sum_{j=1}^{n} \left\langle f_j(T, t) \frac{\partial}{\partial T_j} \prod_{i=1}^{n} T_i^{k_i} \right\rangle + \frac{1}{2} \sum_{i,j=1}^{n} \left\langle (GDG^{\mathrm{T}})_{ij} \frac{\partial^2}{\partial T_1\, \partial T_j} \left(\prod_{i=1}^{n} T_i^{k_i} \right) \right\rangle \tag{24}$$

The solution properties of primary interest are often the moments, particularly the first two moments. Defining $k_i = 1$ in equation (24), one obtains the equation for the first moment:

$$\frac{\mathrm{d}\langle T \rangle}{\mathrm{d}t} = \langle f(T, t) \rangle \tag{25}$$

Analogously to our derivation of the equation for the first moment, we can obtain a set of equations for the second moment by defining a set of k_n's such that

$$\sum_{i=1}^{n} k_i = 2$$

The second moment equation is

$$\frac{\mathrm{d}\langle T_i T_j \rangle}{\mathrm{d}t} = \langle T_i \quad f_j(T, t) \rangle + \langle T_i \quad f_j(T, t) \rangle + 0.5 \langle (GDG^{\mathrm{T}})_{ij} \rangle \tag{26}$$

In general, the set of second-moment equations may involve the first-moment equations. For example, in the previous problem, $n = 2$ (i.e., T_1 and T_2), $\theta = T_1 T_2$, $m = 1, 2$ (i.e., first and second moments). We generate the moment equations by substituting appropriate values for k_i in equation (24). For the first moment (mean value) we generate the two equations by setting

$$k_1 = 1, k_2 = 0: \quad \frac{d\bar{T}_1}{dt} = \frac{\bar{h}_{co}}{c}(\bar{T}_o - \bar{T}_1) + \frac{k}{c}(\bar{T}_2 - \bar{T}_1) + \frac{\bar{Q}_o}{c}$$

$$k_1 = 0, k_2 = 1: \quad \frac{d\bar{T}_2}{dt} = \frac{\bar{h}_{ci}}{c}(T_i - \bar{T}_2) + \frac{k}{c}(\bar{T}_1 - \bar{T}_2)$$

which are the same as the deterministic equations in equation (2). Similarly, the second-moment equations are

$k_1 = 2, k_2 = 0$:

$$\frac{d\overline{T_1^2}}{dt} = -\frac{2}{c}\left(\bar{h}_{co} + k - \frac{1}{2c}\sigma_{hco}^2\right)\overline{T_1^2} + \frac{2k}{c}\overline{T_1 T_2} + \frac{2}{c}(\bar{Q}_o + \bar{h}_{co}\bar{T}_o)\bar{T}_1 + \frac{1}{c^2}(\sigma_{Q_o}^2 + \bar{h}_{co}^2\sigma_T^2 + \bar{T}_o^2\sigma_{hco}^2 - 2\bar{T}_1\bar{T}_o\sigma_{hco}^2)$$

$k_1 = 1, k_2 = 1$:

$$\frac{d\overline{T_1 T_2}}{dt} = \frac{k}{c}\overline{T_1^2} - \frac{1}{c}(\bar{h}_{co} + 2k + h_{ci})\overline{T_1 T_2} + \frac{k}{c}\overline{T_2^2} + \frac{\bar{Q}_o + \bar{h}_{co}\bar{T}_o}{c}\bar{T}_2 + \frac{h_{ci}T_i}{c}\bar{T}_1$$

$k_1 = 0, k_2 = 2$;

$$\frac{d\overline{T_2^2}}{dt} = +\frac{2k}{c}\overline{T_1 T_2} - \frac{2}{c}(k + h_{ci})\overline{T_2^2} + \frac{2h_{ci}}{c}T_i\overline{T_2}$$

The solar radiation, ambient air temperature, and wind velocity are assumed to be independent stochastic processes, distributed as $N(Q_o, \sigma_{Q_o})$, $N(T_o, \sigma_{T_o})$, and $N(v, \sigma_v)$, respectively. The standard deviation of solar radiation, of temperature, and of wind velocity are considered time independent.[4] The probability densities of the outside air temperature and the inside and outside wall surface temperatures at mid-noon are shown in *Figure 1*. As expected, the wall absorbs the fluctuations of random terms as heat penetrates it.

In building load calculations other terms, such as internal heat gain and infiltration, directly affect the heating and cooling loads. To study the effect of these forcing functions on building performance, we model a direct-gain room.

Figure 2 is a schematic of the building system considered in this study: a 3 × 3.5 × 2.5 m direct-gain room with a 5-m² south-facing double-glazed window. A direct-gain solar room is simply a room with a south-facing window. Sunlight entering through the window creates a greenhouse effect sufficient to meet or exceed the current heating requirements of the room. The additional collected energy is stored in the thermal mass to meet later needs (when the sun is not shining). This thermal mass acts as a temporary storage, absorbs heat when it is plentiful, holds a substantial amount until it is needed, then readily gives off sufficient heat to maintain thermal comfort inside the room. The floor and back wall, constructed of concrete 20 cm thick, are discretized in three slices. The network is an eight-node model: one node for room air temperature, six nodes for floor and back wall, and one node for the inside surface of the glazing; see *Figure 2(a)*.

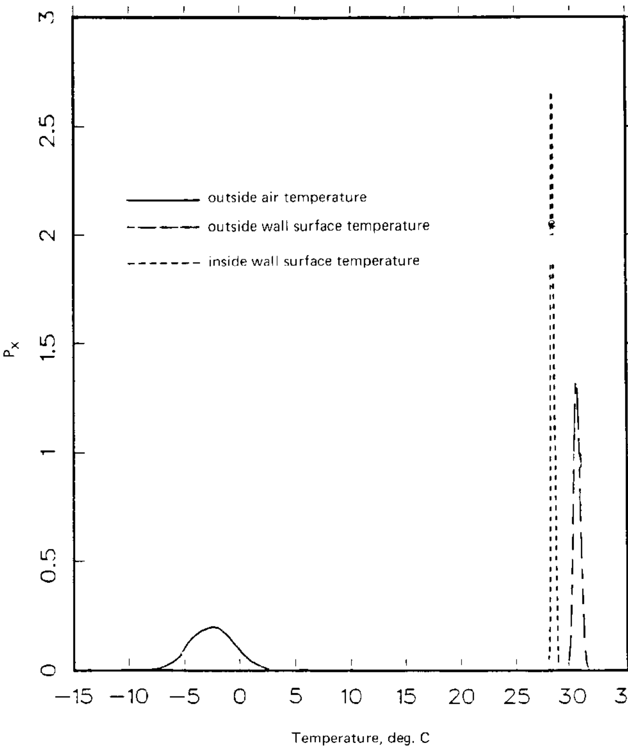

Figure 1 The variation of the probability distribution of temperatures

The analysis contains assumptions that need elaboration. The heat transfer from the floor to the earth and from the back wall to the surrounding environment are neglected. The building enclosure is assumed to behave as a blackbody cavity, and all the solar radiation passing through the glazing is assumed to be absorbed by the thermal storage of floor and back wall.

In this method the number of equations generated increases very rapidly as the number of variables increase. Automatic formulation procedures based on network theoretical techniques have been developed.[21,22] These techniques are well suited for computer implementation. Such procedures require only simple interconnection information, nominal component values such as thermal conductance or thermal mass, and driving functions. The resulting equations are ordinary differential equations with constant coefficients that can be solved by efficient numerical techniques.

The equations are solved by an implicit solution procedure under the following conditions:

a. Geographical location and climatological conditions tions
   latitude = 47. N
   mean daily clearness index $(\bar{K}_T) = 0.52$; $\sigma_{K_T} = 0.17$

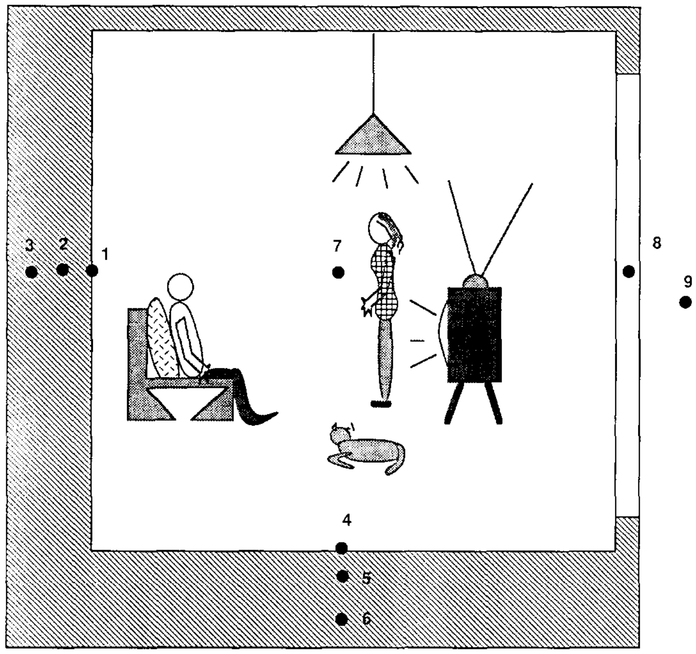

Figure 2 Direct-gain solar room

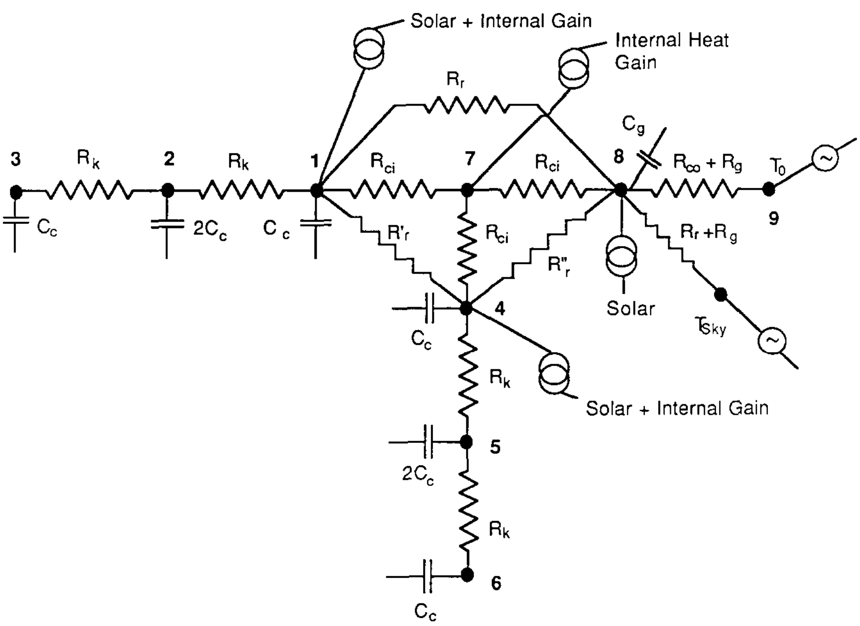

Figure 2a Thermal network representation of direct-gain room

mean outside temperature $(\bar{T}_0) = -5.C°$; $\sigma_T = 0.2C°$
mean wind speed $(\bar{v}_0) = 5$ m/s; $\sigma_{v0} = 2$ m/s

b. The physical properties of concrete
density: $l = 2310$ kg/m^3
specific heat: $C_p = 840$ J/kg · C°
thermal conductivity: $k = 1.73$ W/m · C°

c. Other inputs
internal heat gain: $\dot{Q} = 150$ W; $\sigma_Q = 50$ W
infiltration: $n = 0.5$ air changes per hour; $\sigma_n = 0.5$

The mean hourly $\bar{K}_t$ is obtained from the Collares-Pereira–Rabl[23] relation, and the solar radiation intensity on the wall is estimated from Klucher.[24] The diffuse radiation is obtained from an expression suggested by Orgill and Hollands.[25] A formula of Erbs *et al.*[26] is used to calculate the hourly temperature. The relationship between the outside film coefficient (h_{co}) and wind speed (v_0) is[27]

$$h_{co} = 2.8 + 3v_0$$

The standard deviation of the outside film coefficient can be determined from this expression.

Results

In the deterministic design procedure we are concerned primarily with average conditions, not extremes. Therefore, the design may have insufficient capacity in some of the system components.

As noted earlier, stochastic modelling of building loads provides complementary information to that

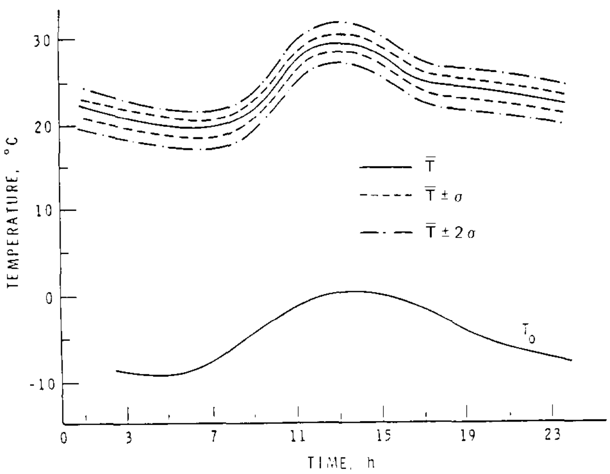

Figure 3 Room air temperature and outside air temperature versus time

obtainable from the deterministic method. This information is required where the parameters that describe radiation, ambient temperature, wind speed, internal heat gain, and infiltration allow random fluctuations. This method provides a tolerance range such that, if the operating systems and related equipment are chosen within that range, the building will be able to cope with anticipated climatic and operational extremes.

The solution of the moment equations provides the statistical properties of the room air temperature. The mean value and the distribution of the room air temperature for one and two standard deviations are shown in *Figure 3*. With this information one designs the heating and cooling system with different levels of confidence. For example, the system can be designed with a 66.7% level of confidence for the range (mean $- \sigma$, mean $+ \sigma$), and with a 95% level of confidence for the range (mean $- 2\sigma$, mean $+ 2\sigma$), and so on. Such complementary information, characteristic of the stochastic nature of the system, would greatly assist a designer in selecting appropriate components to match the desired level of confidence in meeting performance indices such as the load.

Further study shows that the variation in the room air temperature originates mainly from fluctuations of the infiltration, the outside air temperature, and the internal heat gain. The variation in room air temperature is increased from 1.0C° to 1.3C° when the standard deviation of the internal heat gain is varied from 50 to 100 W. Similarly, the effect of the thermal mass of the room and the standard deviation of the number of air changes are studied. When the standard deviation of the number of air changes is increased from 0.25 to 0.5 air changes per hour, the standard deviation of the room air is increased by almost 0.6C°. When the thickness of the floor is changed from 0.5 to 20 cm, the standard deviation of the room air is increased by almost 0.5C°.

The variance of the number of air changes per hour and the internal heat gain are considered to be 0.25 and 2500, respectively. Further research is required to study the statistical properties of these parameters.

Conclusion

A procedure has been developed to model the thermal behaviour of buildings stochastically. This method provides complementary information to that derivable from the deterministic method. This information is based on those parameters that describe the nature of solar radiation, outdoor temperature, internal heat gain, wind speed, and infiltration.

An important advantage of this method is that it enables designers to compare and evaluate alternative designs and take into account the stochastic nature of the system. The most important information from an application viewpoint is the standard deviation at each time point. This knowledge can help us determine the confidence level, which we can then use to calculate the exceedance probability of a variable.

The other advantages of this method are that it simplifies the data-handling procedure; it can use several years of data as a base, which would give a better long-term performance prediction; and the randomness in the input as well as in the coefficients of the model can be considered simultaneously.

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